HW1 - DFA and Regular Languages

COMP361 — Suhas Arehalli

Spring 2025

Instructions

AS with previous homeworks,

- Deadlines are posted on the course website.
- Solutions should be turned in as a typeset pdf, preferably using LaTeX. I recommend a tool like overleaf.
- Assignments are individual, and you should not share solutions with other students. However, you are free to discuss problems within the bounds of the academic integrity policy in the syllabus, and should indicate in your assignment the people you got assistance from (including instructors, peers, and preceptors!).
- Proofread your work before submission, and consult the proof style guide to make sure your submissions are both correct and clear.
- Graded problems will be indicated by an asterisk (*) next to the problem number. You are still expected to solve all of the problems.

Questions

1. *(Sipser 1.6bem)* For each of the following languages, prove that the language is regular by constructing a DFA that recognizes that language, presented as a 5-tuple. For each, the alphabet $\Sigma = \{0, 1\}$. You do **not** need to present a formal proof that the DFA does, in fact, recognize the language.
   (a) $L_1 = \{w \in \Sigma^* \mid w \text{ contains at least three 1s}\}$
   (b) $^*L_2 = \{w \in \Sigma^* \mid w \text{ begins with 0 and has odd length or begins with 1 and has even length}\}$
   (c) $L_3 = \emptyset$
   (d) $L_4 = \{\varepsilon\}$

2. *(Extending Sipser 1.6f)* Recall that in class, we defined $q \xrightarrow{w}_M q'$ to mean that, for a DFA $M = (Q, \Sigma, \delta, q_0, F)$ there exists a sequence of states $r_0, \ldots, r_{|w|} \in Q$ such that (1) $r_0 = q$, (2) $r_i = \delta(r_{i-1}, w_i)$ for all $1 \le i \le n$, and (3) $r_{|w|} = q'$.

   Consider the state diagram for the DFA M_1 in Fig. 1:

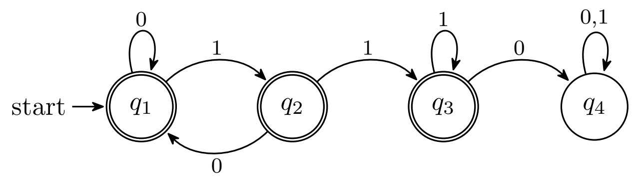

Figure 1: A state diagram for M_1

(a) *Provide a formal definition of the DFA M_1 (i.e., a 5-tuple).

(b) Prove that $q_4 \xrightarrow{w}_{M_1} q_4$ for all $w \in \Sigma^*$.

(c) *Prove that $q' \xrightarrow{110}_{M_1} q_4$ for all $q' \in Q$.

(d) *Use the previous 2 parts to show that if 110 is a substring of w, $q_0 \xrightarrow{w}_{M_1} q_4$, and thus w is rejected by M_1.

(e) Prove that if $q_1 \xrightarrow{w}_{M_1} q_4$, then 110 is a substring of w.

(f) *Use the prior parts to show that M_1 recognizes $A = \{w \in \Sigma^* \mid w \text{ does not contain the substring } 110\}$