HW3 - Preparing for the Pumping Lemma

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Instructions

As with previous homeworks,

- Deadlines are posted on the course website.
- Solutions should be turned in as a typeset pdf, preferably using LaTeX. I recommend a tool like Overleaf.
- Assignments are individual, and you should not share solutions with other students. However, you are free to discuss problems within the bounds of the academic integrity policy in the syllabus, and should indicate in your assignment the people you got assistance from (including instructors, peers, and preceptors!).
- Proofread your work before submission, and consult the proof style guide to make sure your submissions are both correct and clear.
- Graded problems will be indicated by an asterisk (*) next to the problem number. You are still expected to solve all of the problems.

Questions

1. *(Based on Sipser 1.44)* We will show, in a number of steps, that for all $k \geq 1$, there exists a language $A_k \subseteq \{0,1\}^*$ such that a DFA with k states can accept it, but a DFA with $k-1$ states cannot.
   (a) *Consider the language $A_k = \{w \in \Sigma^* \mid |w| \geq k-1\}$, the set of strings using our alphabet of length $k-1$ or greater. Show that for any $k \geq 1$, you can construct a DFA (by providing a 5-tuple) that will recognize A_k that contains exactly k states.
   (b) *Now prove that no DFA with $k-1$ states can recognize A_k.
   ***HINT**: Consider the string $0^{k-1} \in A_k$. Suppose a DFA M with $k-1$ states existed that recognized A_k, and thus accepted 0^{k-1}. Inspect the computation that demonstrates the acceptance of 0^{k-1} and arrive at a contradiction.*
2. *Sipser 1.54/1.55* Let $\Sigma = \{a, b\}$. Consider the language $C_k = L(R)$ defined by the regular expression $R = \Sigma^* a \Sigma^{k-1}$. That is, the language of strings with a the symbol a exactly k places from the end.

(a) Provide a 5-tuple defining an NFA with $k+1$ states that recognizes C_k

(b) Prove that for each $k \geq 1$, no DFA with fewer than 2^k states can recognize C_k.
***HINT**: Construct an argument similar in spirit to your solution to 1b*